Nonlinear Equations

1. Convert the following pseudocodes to program codes, then solve

(a) $f(x) = 2x^3 - 11.7x^2 + 17.7x - 5$ (b) $f(x) = 4e^{-x} - \sinh x$

(c) $f(x) = x^3 - 16x - e^{-x/2}$ (d) $f(x) = e^{-x^2} + x^2 - 4x$

<u>Bisection</u>

```
Read a, b, tol, iteration
fa=f(a)
count=0
E=1
while E>tol and count<iteration
      c=(a+b)/2
      fc=f(c)
      if fa*fc<0 then
            b=c
      else
            a=c
            fa=fc
      endif
      E=|fc|
      count=count+1
endwhile
```

<u>Newton-Raphson</u>

```
Read x0, tol, iteration
      y=f(x0)
      y’=f’(x0)
      E=tol+1
      count=0
While E>tol and count<iteration
      x1=x0-y/y’
      y=f(x1)
      y’=f’(x1)
      Count=Count+1
      E=|y|
      x0=x1
End While
```

<u>Secant</u>

```
Read x0, xm, tol, iteration
      y=f(x0)
      ym=f(xm)
      E=tol+1
      count=0
While E>tol and count<iteration
      x1=x0-y*(x0-xm)/(y-ym)
      ym=y
      xm=x0
      y=f(x1)
      Count=Count+1
      E=|y|
      x0=x1
End While
```

2. Use bisection, Newton-Raphson, and secant methods to find the square root of a positive number “manually” as well as write program code to solve it.

System of Linear Equations

1. Convert the Gauss elimination pseudocodes in slides 27,28 to program codes, then solve

(a) $10x_1 + 2x_2 - x_3 = 27; -3x_1 - 6x_2 + 2x_3 = -61.5; x_1 + x_2 + 5x_3 = -21.5$

(b) $4x_1 + x_2 - x_3 = -2; 5x_1 + x_2 + 2x_3 = 4; 6x_1 + x_2 + x_3 = 6$

(c) $2x_1 - 6x_2 - x_3 = -38; -3x_1 - x_2 + 7x_3 = -34; -8x_1 + x_2 - 2x_3 = -20$

2. Modify the Gauss elimination codes to implement Gauss-Jordan method, then solve problems in 1. as well as find inverse matrices.

3. Modify the Gauss elimination codes to implement LU decomposition, then solve problems in 1. as well as find inverse matrices.

4. Implement the Gauss-Seidel method, then solve problems in 1.

5. Write program codes for finding inverse matrix using Gauss-Jordan method and LU decomposition.

Curve Fitting

1. Use least-squares linear regression to fit a straight line to the data

x	1	2	3	4	5	6	7	8	9	10
y	1	1.5	2	3	4	5	8	10	13	16

Plot the equation together with the data on the same graph.

<u>Note</u>

(i) It is convenient to use "ones" function and vector concatenations to form a matrix.

(ii) Form matrix **A** from vector $\underline{\mathbf{x}}$, then solve $\mathbf{A}^T\mathbf{A}\ \underline{\mathbf{c}} = \mathbf{A}^T\underline{\mathbf{y}}$.

2. Repeat problem 1 using polynomial regression to fit a parabola to the data.

3. Use least-squares regression to find R from the data (V (V), I (mA)) = (1, 1.05), (2, 1.99), (3,3.02), (4, 3.98), (5, 5.01).

4. Fit an exponential model to

X	0.4	0.8	1.2	1.6	2	2.3
Y	800	975	1500	1950	2900	3600

Plot the data and the equation on both standard and semi-logarithmic graph.

5. Write program codes for finding coefficients of interpolating polynomials using

(a) Newton's divided difference (order 1-3) (b) Lagrange's method (order 1-3)

(c) Direct matrix equation (d) Quadratic splines

Then apply to the following data

x	1.6	2	2.5	3.2	4	4.5
f(x)	2	8	14	15	8	2

to find f(2.8) and f(3.5).

6. Suppose the Bessel function $J_1(x)$ is given as follows:

x	1.8	2	2.2	2.4	2.6	2.8
$J_1(x)$	0.5815	0.5767	0.556	0.5202	0.4708	0.4097

Find $J_1(2.1)$, $J_1(2.5)$ (a) using an interpolation and (b) using quadratic spline.

7. Given electricity demand data (source: EGAT) below, use a regression to forecast the future trend.

Year	1980	1985	1990	1995	2000	2005	2010	2015
Consumption [1000 MWh]	2.42	3.88	7.09	12.27	14.92	20.54	24.01	27.35

According to EGAT, the demands in years 2016, 2017 were 29,618 and 28,578 MWh, respectively.

8. Given world population data (source: Worldometers) below, use a regression to forecast the future trend.

Year	1965	1970	1975	1980	1985	1990	1995	2000	2005	2010	2015
Population [billions]	3.34	3.7	4.08	4.46	4.87	5.33	5.75	6.15	6.54	6.96	7.38

Numerical Differentiation

1. Write the program codes to compute the first derivative of a function using

(i) Forward difference (ii) Backward difference (iii) Central difference

Then apply them to the following functions:

(a) $f(x) = x^3 + 4x - 15; x = [-2,2]; h = \{0.5, 0.25, 0.125\}$

(b) $f(x) = x^2 \cos x; x = [0,4]; h = \{0.5, 0.25, 0.125\}$

(c) $f(x) = \tan(x/3); x = [0,4]; h = \{0.5, 0.25, 0.125\}$

(d) $f(x) = \sin(0.5\sqrt{x})/x; x = [0.5,2]; h = \{0.5, 0.25, 0.125\}$

(e) $f(x) = e^x + x; x = [-2,2]; h = \{0.5, 0.25, 0.125\}$

Also compare with the analytic results.

2. Repeat problem 1 using Richardson extrapolation.

3. Write a program code to compute the second derivative of a function, then apply it to the functions in problem 1 and compare results with the analytic ones.

4. Given the following measurement result regarding an unknown resistor, find the *voltage-dependent* resistance.

V [V]	1	1.5	2	2.5	3	3.5	4
I [mA]	1.1	1.55	2.0	2.45	2.9	3.35	3.8

5. Given the following voltage across the capacitor versus time characteristics of a capacitor of capacitance 10 μF, determine the current flowing through this capacitor.

Time [s]	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Voltage [mV]	0	.25	.50	.75	.85	.92	.95	.97	.98	.99	1.0

6. Suppose a battery charging characteristics is given below, estimate the current assuming electric loss is negligible. Here, assume the battery capacity is 3,000 mAh, and it is designed to be used with 5 V.

Time [min]	0	15	30	45	60	75	90	105	120	135	150
Capacity [%]	0	25	50	75	85	92	95	97	98	99	100

Numerical Integration

1. Write program codes to evaluate integrals using
(i) midpoint rule (ii) trapezoidal rule (iii) Simpson 1/3 rule
Then apply them to evaluate

(a) $\int_0^{\pi/2}(1+2\cos x)dx$ (b) $\int_0^3(1-e^{-x})dx$

(c) $\int_0^1(x^2\sinh x+\tan^{-1}x)dx$ (d) $\int_1^2(x+1/x)^2dx$

Compare the results with the analytic ones, then estimate the number of segments n such that the error becomes less than 10^{-6} for each method.

2. Write the program code for the "recursive" trapezoidal rule, then use it to implement the Romberg method. Also, apply the codes to the integrals in 1.

3. Write the program code for the Gauss quadrature with n = 2, 3, 4.

4. Apply all the codes developed above to evaluate the following integrals numerically

(a) $\int_0^2\frac{e^x\sin x}{1+x^2}dx$ (b) $\frac{2}{\sqrt{\pi}}\int_0^1 e^{-x^2}dx$

(c) $\int_1^2\frac{\sin x}{x}dx$ (d) $\int_1^2\frac{1-\cos x}{x}dx$

Here, choose an appropriate n for midpoint, trapezoidal, and Simpson methods.

5. Determine the RMS value of the following current:

(a) $i(t)$ = $10\sin 2\pi t$ (b) $i(t)$ = $10\,e^{-0.1t}\sin 2\pi t$

(c) $i(t)=\begin{cases}5\sin 10\pi t & 0\le t\le T/2\\ 0 & T/2\le t\le T\end{cases}$ (d) $i(t)=\begin{cases}5e^{-0.5t}\sin 10\pi t & 0\le t\le T/2\\ 0 & T/2\le t\le T\end{cases}$

6. Suppose applying a constant voltage of 1V to a capacitor gives rise to the following result, estimate the capacitance from this data.

Time [s]	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Current [mA]	10	7.8	6.1	4.7	3.7	2.9	2.2	1.7	1.4	1.1	0.9

7. The table below shows the result when applying a constant voltage of 12V to a load. Find the average power consumed in this load.

Time [hour]	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
Current [mA]	10	11	8	7	15	13	9	10	14	11	6

Ordinary Differential Equations

1. Write program codes for implementing the following methods:

(i) Euler (ii) mid-point (iii) RK2 (iv) RK4

to solve first-order differential equations.

2. Apply codes in 1 to solve the following problems:

(i) Let R = 1 kΩ, C = 1 μF, find

(a) $v_C(t)$ for a series RC circuit with $v_C(0^-)=0$ and voltage source $V_s(0^+) = u(t)$ V.

(b) $v_C(t)$ for a parallel RC circuit with $v_C(0^-)=0$ and current source $I_s(0^+) = u(t)$ A.

(ii) Let R = 1 Ω, L = 1 mH, find

(c) $i_L(t)$ for a series RL circuit with $i_L(0^-)=0$ and voltage source $V_s(0^+) = u(t)$.

(d) $i_L(t)$ for a parallel RL circuit with $i_L(0^-)=0$ and current source $I_s(0^+) = u(t)$.

(iii) Repeat problem (i) with source changed to triangular pulse of height 1 and width 1 ms.

(iv) Repeat problem (ii) with source changed to triangular pulse of height 1 and width 1 ms.

3. Write program codes for implementing the following methods:

(i) Euler (ii) RK2

to solve second-order differential equations.

4. Apply codes in 3 to solve the following problems:

(i) Let R = 2 kΩ, L = 0.1 H, C = .1 μF, find

$v_C(t)$ for a series RLC circuit with $v_C(0^-)=0$, $i_C(0^-) = 0$ and voltage source $V_s(0^+) = u(t)$

Then repeat the problem with C changed to 1 μF and 10 nF, respectively.

(ii) Repeat problem (i) with source changed to triangular pulse of height 1 and width 4 ms.

(iii) Let R = 0.8 kΩ, L = 0.1 H, C = .1 μF, find

$i_L(t)$ for a parallel RLC circuit with $v_L(0^-)=0$, $i_L(0^-) = 0$ and current source $I_s(0^+) = u(t)$.

Then repeat the problem with C changed to 1 μF and 10 nF, respectively.

(iv) Repeat problem (iii) with source changed to triangular pulse of height 1 and width 4 ms.

NOTE:

1. $u(t-a)$ denotes the unit step function given by:

$$u(t-a) = \begin{cases} 1 & t \geq a \\ 0 & t < a \end{cases}$$

2. Continue computations until systems reach “steady” states.

Eigenvalue Problem (Power Method)

1. Implement the power method to find the largest eigenvalue and its corresponding eigenvector.

2. Apply the code in 1 to the following matrices:

(a) $\begin{bmatrix} 11 & -8 & 4 \\ -8 & -1 & -2 \\ 4 & -2 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 2 & -2 \\ -2 & 8 & 1 \\ 2 & 4 & -4 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 0 & 1 \\ 1 & -1 & 0 \\ 3 & 0 & 4 \end{bmatrix}$

Here, try the method with various choices of initial vector.

3. Repeat problem 2 with

(a) $\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 0 & 3 & 1 \\ 1 & 3 & -4 & -2 \\ 0 & 1 & -2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 2 & 0 & 1 \\ 2 & 0 & 5 & -1 \\ 0 & 5 & 2 & 1 \\ 1 & -1 & 1 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 2 & -1 & 0 \\ 0 & -1 & 3 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}$