Chapter 3

Signal Degradation in Optical Fibers
Outline

1. Attenuation
   - Absorption and Scattering
   - Bending
   - Core, cladding losses

2. Signal Distortion
   - Intramodal (or Chromatic) dispersion: Material & waveguide dispersions
   - Modal dispersion

3. Characteristics of single-mode fibers, e.g.,
   - Refractive-index profiles
   - Cut-off wavelengths
Signal Attenuation & Distortion in Optical Fibers

• What are the loss or signal attenuation mechanism in a fiber?
• Why & to what degree do optical signals get distorted as they propagate down a fiber?

• Signal attenuation (fiber loss) largely determines the maximum repeaterless separation between optical transmitter & receiver.
• Signal distortion cause that optical pulses to broaden as they travel along a fiber, the overlap between neighboring pulses, creating errors in the receiver output, resulting in the limitation of information-carrying capacity of a fiber.
Attenuation (fiber loss)

- Power loss along a fiber:

\[ P(z) = P(0)e^{-\alpha_p z} \quad [3-1] \]

- The parameter \( \alpha_p \) is called fiber attenuation coefficient in units of for example [1/km] or [nepers/km]. A more common unit is [dB/km] that is defined by:

\[
\alpha_{[\text{dB/km}]} = \frac{10}{l} \log \left( \frac{P(0)}{P(l)} \right) = 4.343 \alpha_p [1/\text{km}] \quad [3-2]
\]
Fiber loss in dB/km

\[ P(l)[\text{dBm}] = P(0)[\text{dBm}] - \alpha[\text{dB/km}] \times l[\text{km}] \]  \hspace{1cm} [3-3]

- Where [dBm] or dB milliwatt is 10\log(P[\text{mW}]).
Optical fiber attenuation vs. wavelength

Absorption

• Absorption is caused by three different mechanisms:
  1- Impurities in fiber material: from transition metal ions (must be in order of parts per billion (ppb)) & particularly from OH ions with absorption peaks at wavelengths 2700 nm, 1400 nm, 950 nm & 725nm.
  2- Intrinsic absorption (fundamental lower limit): electronic absorption band (UV region) & atomic bond vibration band (IR region) in basic SiO$_2$.
  3- Atomic defects: typically relatively small except exposed to nuclear radiation level (e.g., inside a nuclear reactor, nuclear explosion)
Scattering Loss

- Scattering losses are due to microscopic variations in the material density, compositional fluctuations (inclusion of GeO$_2$, P$_2$O$_5$, etc.), structural inhomogeneities and defects.
- The first two effects give rise to refractive-index variations.
- The refractive-index variations cause Rayleigh scattering, which is the same phenomenon as the scattering of light in the atmosphere giving rise to blue sky.
- Rayleigh scattering is proportion to $\lambda^{-4}$.
- The structural inhomogeneities and defects (created during fabrication) can cause scattering of light out of fiber. These defects include trapped gas bubbles, unreacted starting materials and crystallized regions.
- These defects are minimized by improving preform manufacturing methods and this scattering is negligible compared to the Rayleigh scattering.
Absorption & scattering losses in fibers

[Graph showing absorption and scattering losses in fibers with wavelengths and photon energies on the x-axis and loss in dB/km on the y-axis.]

Typical spectral absorption & scattering attenuations for a single mode-fiber

Bending Loss (Macrobending & Microbending)

**Macrobending Loss:** The curvature of the bend is much larger than fiber diameter. Lightwave suffers severe loss due to radiation of the evanescent field in the cladding region. As the radius of the curvature decreases, the loss increases exponentially until it reaches at a certain critical radius. For any radius a bit smaller than this point, the losses suddenly become extremely large. Higher order modes radiate away faster than lower order modes.
Microbending Loss

- **Microbending Loss:** microscopic bends of the fiber axis that can arise when the fibers are incorporated into cables. The power is dissipated through the microbended fiber, because of the repetitive coupling of energy between guided modes & the leaky or radiation modes in the fiber.

Dispersion in Optical Fibers

- **Dispersion**: Any phenomenon in which the velocity of propagation of any electromagnetic wave is wavelength dependent.

- In communication, dispersion is used to describe any process by which any electromagnetic signal propagating in a physical medium is degraded because the various wave characteristics (i.e., frequencies) of the signal have different propagation velocities within the physical medium.

- There are 3 dispersion types in the optical fibers, in general:

  1- Material Dispersion
  2- Waveguide Dispersion
  3- Modal Dispersion -> Intermodal delay

  Material & waveguide dispersions are main causes of Intramodal Dispersion.
Group Velocity

Wave Velocities:

1- **Plane wave velocity**: For a plane wave propagating along $z$-axis in an unbounded homogeneous region of refractive index $n_1$, which is represented by $\exp(j\omega t - jk_1 z)$, the velocity of constant phase plane is:

$$v = \frac{\omega}{k_1} = \frac{c}{n_1}$$

2- **Modal wave phase velocity**: For a modal wave propagating along $z$-axis represented by $\exp(j\omega t - j\beta z)$, the velocity of constant phase plane is:

$$v_p = \frac{\omega}{\beta}$$

3- For transmission system operation the most important & useful type of velocity is the **group velocity**, $V_g$. This is the actual velocity which the signal information & energy is traveling down the fiber. It is always less than the speed of light in the medium. The observable delay experiences by the optical signal waveform & energy, when traveling a length of $l$ along the fiber is commonly referred to as **group delay**.
Group Velocity & Group Delay

• The group velocity is given by:

\[ V_g = \frac{d\omega}{d\beta} \]

• The group delay is given by:

\[ \tau_g = \frac{l}{V_g} = l \frac{d\beta}{d\omega} \]

• It is important to note that all above quantities depend both on frequency & the propagation mode. In order to see the effect of these parameters on group velocity and delay, the following analysis would be helpful.
Input/Output signals in Fiber Transmission System

- The optical signal (complex) waveform at the input of fiber of length \( l \) is \( f(t) \). The propagation constant of a particular modal wave carrying the signal is \( \beta(\omega) \). Let us find the output signal waveform \( g(t) \).

\( \Delta \omega \) is the optical signal bandwidth.

\[
f(t) = \int_{\omega_c - \Delta \omega}^{\omega_c + \Delta \omega} \tilde{f}(\omega) e^{j\omega t} \, d\omega
\]

\[
g(t) = \int_{\omega_c - \Delta \omega}^{\omega_c + \Delta \omega} \tilde{f}(\omega) e^{j\omega t - j\beta(\omega)l} \, d\omega
\]
If $\Delta \omega << \omega_c$

$$
\beta(\omega) = \beta(\omega_c) + \frac{d\beta}{d\omega}\bigg|_{\omega=\omega_c} (\omega - \omega_c) + \frac{1}{2} \frac{d^2 \beta}{d\omega^2}\bigg|_{\omega=\omega_c} (\omega - \omega_c)^2 + ...
$$

$$
g(t) = \int_{\omega_c - \Delta \omega/2}^{\omega_c + \Delta \omega/2} \tilde{f}(\omega)e^{j\omega t - j\beta(\omega)t} d\omega \approx \int_{\omega_c - \Delta \omega/2}^{\omega_c + \Delta \omega/2} \tilde{f}(\omega)e^{j\omega t - j[\beta(\omega_c) + \frac{d\beta}{d\omega}\bigg|_{\omega=\omega_c} (\omega - \omega_c)]t} d\omega
$$

$$
\approx e^{-j\beta(\omega_c)t} \int_{\omega_c - \Delta \omega/2}^{\omega_c + \Delta \omega/2} \tilde{f}(\omega)e^{j\omega(t - l\frac{d\beta}{d\omega}\bigg|_{\omega=\omega_c})} d\omega
$$

$$
= e^{-j\beta(\omega_c)l} f(t - l\frac{d\beta}{d\omega}\bigg|_{\omega=\omega_c}) = e^{-j\beta(\omega_c)l} f(t - \tau_g)
$$

$$
\tau_g = l\frac{d\beta}{d\omega}\bigg|_{\omega=\omega_c} = \frac{l}{V_g}
$$
Intramodal Dispersion

- As we have seen from Input/output signal relationship in optical fiber, the output is proportional to the delayed version of the input signal, and the delay is inversely proportional to the group velocity of the wave. Since the propagation constant, \( \beta(\omega) \), is frequency dependent over band width \( \Delta \omega \) sitting at the center frequency \( \omega_c \), at each frequency, we have one propagation constant resulting in a specific delay time. As the output signal is collectively represented by group velocity & group delay this phenomenon is called intramodal dispersion or Group Velocity Dispersion (GVD). This phenomenon arises due to a finite bandwidth of the optical source, dependency of refractive index on the wavelength and the modal dependency of the group velocity.

- In the case of optical pulse propagation down the fiber, GVD causes pulse broadening, leading to Inter Symbol Interference (ISI).
A measure of information capacity of an optical fiber for digital transmission is usually specified by the **bandwidth-distance product** (BW×distance) in GHz·km. For multi-mode step index fiber this quantity is about 20 MHz·km, for graded index fiber is about 2.5 GHz·km & for single mode fibers are higher than 10 GHz·km.
How to characterize dispersion?

• Group delay per unit length can be defined as:

\[
\frac{\tau_g}{L} = \frac{1}{d\omega} \frac{d\beta}{dk} = \frac{1}{c} \frac{d\beta}{dk} = \frac{-\lambda^2}{2\pi c} \frac{d\beta}{d\lambda}
\]

• If the spectral width of the optical source is not too wide, then the delay difference per unit wavelength along the propagation path is approximately \( \frac{d\tau_g}{d\lambda} \). For spectral components which are \( \delta\lambda \) apart, symmetrical around center wavelength, the total delay difference \( \delta\tau \) over a distance \( L \) is:

\[
\delta\tau = \left| \frac{d\tau_g}{d\lambda} \right| \delta\lambda = -\frac{L}{2\pi c} \left( 2\lambda \frac{d\beta}{d\lambda} + \lambda^2 \frac{d^2\beta}{d\lambda^2} \right) \delta\lambda
\]

\[
= \left| \frac{d\tau}{d\omega} \right| \delta\omega = \frac{d}{d\omega} \left( \frac{L}{V_g} \right) \delta\omega = L \left( \frac{d^2\beta}{d\omega^2} \right) \delta\omega
\]
• \( \beta_2 \equiv \frac{d^2 \beta}{d\omega^2} \) is called \textbf{GVD parameter}, and shows how much a light pulse broadens as it travels along an optical fiber. The more common parameter is called \textbf{Dispersion}, and can be defined as the delay difference per unit length per unit wavelength as follows:

\[
D = \frac{1}{L} \frac{d\tau_g}{d\lambda} = \frac{d}{d\lambda} \left( \frac{1}{V_g} \right) = -\frac{2\pi c}{\lambda^2} \beta_2
\]

• In the case of optical pulse, if the spectral width of the optical source is characterized by its rms value of the Gaussian pulse \( \sigma_\lambda \) [nm], the pulse spreading over the length of L, \( \sigma_g \) [ps] can be well approximated by:

\[
\sigma_g \approx \left| \frac{d\tau_g}{d\lambda} \right| \sigma_\lambda = DL\sigma_\lambda
\]

• \( D \) has a typical unit of [ps/(nm.km)].
All excitation sources are inherently non-monochromatic and emit within a spectrum, $^{2} \lambda$, of wavelengths. Waves in the guide with different free space wavelengths travel at different group velocities due to the wavelength dependence of $n_1$. The waves arrive at the end of the fiber at different times and hence result in a broadened output pulse.

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Material Dispersion

- The refractive index of the material varies as a function of wavelength, \( n(\lambda) \)
- Material-induced dispersion for a plane wave propagation in homogeneous medium of refractive index \( n \):
  \[
  \tau_{mat} = L \frac{d\beta}{d\omega} = -\frac{\lambda^2}{2\pi c} L \frac{d\beta}{d\lambda} = -\frac{\lambda^2}{2\pi c} L \frac{d}{d\lambda} \left[ \frac{2\pi}{\lambda} n(\lambda) \right] 
  = \frac{L}{c} \left( n - \lambda \frac{dn}{d\lambda} \right) 
  
  \]
- The pulse spread due to material dispersion is therefore:
  \[
  \sigma_g \approx \left| \frac{d\tau_{mat}}{d\lambda} \right| \sigma_\lambda = \frac{L\sigma_\lambda}{c} \left| \lambda \frac{d^2n}{d\lambda^2} \right| = L\sigma_\lambda \left| D_{mat}(\lambda) \right| 
  
  \]

\( D_{mat}(\lambda) \) is material dispersion
Material Dispersion Diagrams

Waveguide Dispersion

- Waveguide dispersion is due to the dependency of the group velocity of the fundamental mode as well as other modes on the $V$ number, (see Fig 2-18 of the textbook). In order to calculate waveguide dispersion, we consider that $n$ is not dependent on wavelength. Defining the normalized propagation constant $b$ as:

$$b = \frac{\beta^2 / k^2 - n_2^2}{n_1^2 - n_2^2} \approx \frac{\beta / k - n_2}{n_1 - n_2}$$

- solving for propagation constant:

$$\beta \approx n_2 k (1 + b \Delta); \Delta = (n_1 - n_2) / n_1$$

- Using $V$ number:

$$V = ka(n_1^2 - n_2^2)^{1/2} \approx kan_1 \sqrt{2\Delta}$$
Waveguide Dispersion

- Delay time due to waveguide dispersion can then be expressed as:

\[
\tau_{wg} = \frac{L}{c} \frac{d\beta}{dk} = \frac{L}{c} \left[ n_2 + n_2 \Delta \frac{d(kb)}{dk} \right] \approx \frac{L}{c} \left[ n_2 + n_2 \Delta \frac{d(Vb)}{dV} \right]
\]

Waveguide dispersion in single mode fibers

- For single mode fibers, waveguide dispersion is in the same order of material dispersion. The pulse spread can be well approximated as:

\[
\sigma_{wg} \approx \left| \frac{d\tau_{wg}}{d\lambda} \right| \sigma_{\lambda} = L\sigma_{\lambda} \left| D_{wg}(\lambda) \right| = -\frac{n_2 L\Delta\sigma_{\lambda}}{c\lambda} V \frac{d^2(Vb)}{dV^2}
\]

![Image](image_url)

**FIGURE 3.15**

The waveguide parameter and its derivatives \( \frac{d^2(Vb)}{dV^2} \) and \( V \frac{d^2(Vb)}{dV^2} \) plotted as a function of \( V \) number for the HE_{21} mode.

Modal Dispersion

• In multi-mode fiber, the pulse spread due to the modal dispersion can be approximated by ray tracing as

\[ \sigma_{\text{modal}} \approx \tau_{\text{max}} - \tau_{\text{min}} \]

where \( \tau_{\text{max}}, \tau_{\text{min}} \) denote the maximum, minimum group delays, respectively. Since

\[ \tau_{\text{max}} = L / (v_p \sin \phi_c) ; \tau_{\text{min}} = L / v_p ; v_p = c / n_1 \]

It can be shown that

\[ \sigma_{\text{modal}} \approx \frac{L n_1}{c} \left( \frac{n_1}{n_2} - 1 \right) = \frac{L n_1^2}{cn_2} \Delta \]

In general, \( \sigma_{\text{modal}} < 1/B \) is required, thus bit rate-distance product:

\[ BL < \frac{n_2 c}{n_1^2 \Delta} \]
Suppose that the core refractive index has different values along two orthogonal directions corresponding to electric field oscillation direction (polarizations). We can take \( x \) and \( y \) axes along these directions. An input light will travel along the fiber with \( E_x \) and \( E_y \) polarizations having different group velocities and hence arrive at the output at different times.

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Polarization Mode dispersion

- The effects of fiber-birefringence on the polarization states of an optical are another source of pulse broadening. **Polarization mode dispersion** (PMD) is due to slightly different velocity for each polarization mode because of the lack of perfectly symmetric & anisotropicity of the fiber. If the group velocities of two orthogonal polarization modes are $v_{gx}$ and $v_{gy}$ then the differential time delay $\Delta \tau_{PMD}$ between these two polarization over a distance $L$ is

$$\Delta \tau_{PMD} = \left| \frac{L}{v_{gx}} - \frac{L}{v_{gy}} \right|$$

- The rms value of the differential group delay can be approximated as:

$$\langle \Delta \tau_{PMD} \rangle \approx D_{PMD} \sqrt{L}$$
**Chromatic & Total Dispersion**

- Chromatic dispersion includes the material & waveguide dispersions.

\[ D_{ch}(\lambda) \approx |D_{mat} + D_{wg}| \]

\[ \sigma_{ch} = D_{ch}(\lambda)L\sigma_\lambda \]

- Total dispersion is the sum of chromatic, polarization dispersion and other dispersion types and the total rms pulse spreading can be approximately written as:

\[ D_{total} \approx |D_{ch} + D_{pol} + \ldots| \]

\[ \sigma_{total} = D_{total}L\sigma_\lambda \]
Fact 1) Minimum distortion at wavelength about 1300 nm for single mode silica fiber.
Fact 2) Minimum attenuation is at 1550 nm for single mode silica fiber.
Strategy: shifting the zero-dispersion to longer wavelength for minimum attenuation and dispersion.
Optimum single mode fiber & distortion/attenuation characteristics

Fact 1) Minimum distortion at wavelength about 1300 nm for single mode silica fiber.
Fact 2) Minimum attenuation is at 1550 nm for single mode silica fiber.

Strategy: shifting the zero-dispersion to longer wavelength for minimum attenuation and dispersion by modifying waveguide dispersion by changing from a simple step-index core profile to more complicated profiles. There are four major categories to do that:

1- 1300 nm optimized single mode step-fibers: matched cladding (mode diameter 9.6 micrometer) and depressed-cladding (mode diameter about 9 micrometer)
2- Dispersion shifted fibers.
3- Dispersion-flattened fibers.
4- Large-effective area (LEA) fibers (less nonlinearities for fiber optical amplifier applications, effective cross section areas are typically greater than 100 $\mu m^2$).
Representative cross sections of index profiles for (a) 1310-nm-optimized, (b) dispersion-shifted, (c) dispersion-flattened, and (d) large-effective-core-area fibers.
Single mode fiber dispersion

- Material dispersion
- Standard single-mode (1300-nm-optimized)
- Dispersion-flattened

Dispersion [ps/(nm km)]

Wavelength (nm) (a)

1300 1400 1500 1600

Single mode fiber dispersion

Dispersion \([\text{ps/(nm} \cdot \text{km)}]\)

- 1300-nm-optimized
- Dispersion-flattened
- Dispersion-shifted

Wavelength (nm)

Single mode Cut-off wavelength & Dispersion

- Fundamental mode is $\text{HE}_{11}$ or $\text{LP}_{01}$ with $V=2.405$ and $\lambda_c = \frac{2\pi a}{V} \sqrt{n_1^2 - n_2^2}$
- Dispersion:

$$D(\lambda) = \frac{d\tau}{d\lambda} \approx D_{\text{mat}}(\lambda) + D_{\text{wg}}(\lambda)$$

$$\sigma = D(\lambda)L\sigma_{\lambda}$$

- For non-dispersion-shifted fibers (1270 nm – 1340 nm)
- For dispersion shifted fibers (1500 nm- 1600 nm)
Dispersion for non-dispersion-shifted fibers
(1270 nm – 1340 nm)

\[ \tau(\lambda) = \tau_0 + \frac{S_0}{8} \left( \lambda - \frac{\lambda_0^2}{\lambda} \right)^2 \]

- \( \tau_0 \) is relative delay minimum at the zero-dispersion wavelength \( \lambda_0 \), and \( S_0 \) is the value of the dispersion slope in ps/(nm\(^2\)·km).

\[ S_0 = S(\lambda_0) = \left. \frac{dD}{d\lambda} \right|_{\lambda=\lambda_0} \]

\[ D(\lambda) = \frac{\lambda S_0}{4} \left[ 1 - \left( \frac{\lambda_0}{\lambda} \right)^4 \right] \]
\[ \tau(\lambda) = \tau_0 + \frac{S_0}{2} (\lambda - \lambda_0)^2 \]

\[ D(\lambda) = (\lambda - \lambda_0)S_0 \]

Example of dispersion Performance curve for Set of SM-fiber
Example of BW vs wavelength for various optical sources for SM-fiber.
MFD

![Graph showing mode-field diameter (MFD) against wavelength. The graph includes three curves labeled: 1300-nm-optimized, Dispersion-shifted, and Dispersion-flattened.]
Bending Loss

Added loss due to macrobending

Added loss due to microbending

Basic fiber loss

Loss (dB/km)

Wavelength (nm)

Bending effects on loss vs MFD

\begin{figure}[h]
\centering
\subfigure[Macrobending (1-inch loop)]{
\begin{tikzpicture}
\begin{axis}[
width=0.45\textwidth,
height=0.35\textwidth,
xlabel=Mode-field diameter ($\mu$m),
ylabel=Loss (dB/km),
xmin=8, xmax=12,
ymin=0, ymax=2,
]
\addplot[black, thick, mark=none, domain=8:12] {0.5*(x-10));
\addplot[black, thick, mark=none, domain=8:12] {1.5*(x-10)};
\node at (axis cs:10,1) {Microbending ($L_c = 300 \mu$m, rms = 2 nm)};
\end{axis}
\end{tikzpicture}
}
\hspace{1cm}
\subfigure[Macrobending (1-inch loop)]{
\begin{tikzpicture}
\begin{axis}[
width=0.45\textwidth,
height=0.35\textwidth,
xlabel=Mode-field diameter ($\mu$m),
ylabel=Loss (dB/km),
xmin=8, xmax=12,
ymin=0, ymax=2,
]
\addplot[black, thick, mark=none, domain=8:12] {0.5*(x-10) + 0.5};
\addplot[black, thick, mark=none, domain=8:12] {1.5*(x-10) + 0.5};
\node at (axis cs:10,1) {Microbending ($L_c = 300 \mu$m, rms = 2 nm)};
\end{axis}
\end{tikzpicture}
}
\caption{Bending effects on loss vs MFD}
\end{figure}

Bend loss versus bend radius: Finite vs Infinite Cladding

\[ a = 3.6 \mu m; \quad b = 60 \mu m \]
\[ \Delta = 3.56 \times 10^{-3}; \quad \frac{n_3 - n_2}{n_2} = 0.07 \]