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PROBLEMS

5.1. Derive
(a) (5-18a)–(5-18c) using (5-17) and (3-2a)
(b) (5-19a)–(5-19b) using (5-18a)–(5-18c)

5.2. Write the fields of an infinitesimal linear magnetic dipole of constant current I_m, length l, and positioned along the z-axis. Use the fields of an infinitesimal electric dipole, (4-8a)–(4-10c), and apply the principle of duality. Compare with (5-20a)–(5-20d).

5.3. A circular loop, *of loop radius* $\lambda/30$ *and wire radius* $\lambda/1000$, is used as a transmitting/receiving antenna in a back-pack radio communication system at *10 MHz*. The wire of the loop is made of copper with a conductivity of 5.7×10^7 S/m. Assuming the antenna is *radiating in free space*, determine the
(a) radiation resistance of the loop;

(b) loss resistance of the loop (*assume that its value is the same as if the wire were straight*);
(c) input resistance;
(d) input impedance;
(e) radiation efficiency.

5.4. A small circular loop with a uniform current distribution, and with its classical omnidirectional pattern, is used as a receiving antenna. Determine the maximum directivity (*dimensionless* and *in dB*) using:
(a) Exact method.
(b) An approximate method appropriate for this pattern. Specify the method used.
(c) Another approximate method appropriate for this pattern. Specify the method used.

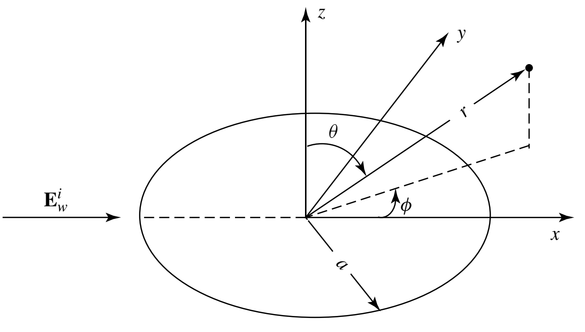
Hint: *For the approximate methods, the word omnidirectional is a clue.*

5.5. A *N-turn* resonant circular loop *with a uniform current distribution* and with a *circumference of* $\lambda/4$, is fed by a lossless balanced twin-lead transmission line with a characteristic impedance of 300 ohms. *Neglecting proximity effects*, determine the
(a) *closest integer number* of turns so that the input impedance is nearly 300 ohms;
(b) input impedance of the antenna;
(c) reflection coefficient;
(d) VSWR inside the transmission line.

5.6. A small circular loop with circumference $C < \lambda/20$ is used as a receiving antenna. A uniform plane wave traveling along the x-axis and toward the positive (+) x direction (as shown in the figure), whose electric field is given by

$$\mathbf{E}_w^i = (\hat{\mathbf{a}}_y + 2\hat{\mathbf{a}}_z)e^{-jkx}$$

is incident upon the antenna. Determine the



(a) polarization of the incident wave. Justify your answer.
(b) axial ratio of the polarization ellipse of the incident wave.
(c) polarization of the loop antenna toward the x-axis.

(d) polarization loss factor (*dimensionless* and *in dB*).

(e) maximum power at *1 GHz* that can be delivered to a load connected to the antenna, if the power density of the above incident wave is $5\ mwatts/cm^2$. Assume no other losses.

Hint: $\hat{\mathbf{a}}_\phi = -\hat{\mathbf{a}}_x \sin\phi + \hat{\mathbf{a}}_y \cos\phi$

5.7. Find the radiation efficiency of a single-turn and a four-turn circular loop each of radius $\lambda/(10\pi)$ and operating at 10 MHz. The radius of the wire is $10^{-3}\lambda$ and the turns are spaced $3 \times 10^{-3}\lambda$ apart. Assume the wire is copper with a conductivity of 5.7×10^7 S/m, and the antenna is radiating into free-space.

5.8. Find the power radiated by a small loop by forming the average power density, using (5-27a)–(5-27c), and integrating over a sphere of radius r. Compare the answer with (5-23b).

5.9. For a small loop of constant current, derive its far-zone fields using (5-17) and the procedure outlined and relationships developed in Section 3.6. Compare the answers with (5-27a)–(5-27c).

5.10. A single-turn resonant circular loop with a $\lambda/8\pi$ radius is made of copper wire with a wire radius of $10^{-4}\lambda/2\pi$ and conductivity of 5.7×10^7 S/m. For a frequency of 100 MHz, determine, *assuming uniform current*, the

(a) radiation efficiency (assume the wire is straight);

(b) *maximum gain* of the antenna (*dimensionless* and *in dB*).

5.11. Design a lossless resonant circular loop operating at 10 MHz so that its single-turn radiation resistance is 0.73 ohms. The resonant loop is to be connected to a matched load through a balanced "twin-lead" 300-ohm transmission line.

(a) Determine the radius of the loop (in meters and wavelengths).

(b) To minimize the matching reflections between the resonant loop and the 300-ohm transmission line, determine the closest number of integer turns the loop must have.

(c) For the loop of part b, determine the maximum power that can be expected to be delivered to a receiver matched load if the incident wave is polarization matched to the lossless resonant loop. The power density of the incident wave is 10^{-6}watts/m^2.

5.12. A resonant six-turn loop of *closely spaced turns* is operating at 50 MHz. The radius of the loop is $\lambda/30$, and the loop is connected to a 50-ohm transmission line. The radius of the wire is $\lambda/300$, its conductivity is $\sigma = 5.7 \times 10^7$ S/m, and the spacing between the turns is $\lambda/100$. Determine the

(a) directivity of the antenna (in dB)

(b) radiation efficiency taking into account the proximity effects of the turns

(c) reflection efficiency

(d) gain of the antenna (in dB)

5.13. Find the radiation efficiency (in percent) of an eight-turn circular-loop antenna operating at 30 MHz. The radius of each turn is $a = 15$ cm, the radius of the wire is $b = 1$ mm, and the spacing between turns is $2c = 3.6$ mm. Assume the wire is copper ($\sigma = 5.7 \times 10^7$ S/m), and the antenna is radiating into free-space. Account for the *proximity effect*.

5.14. A very small circular loop of radius $a(a < \lambda/6\pi)$ and constant current I_0 is symmetrically placed about the origin at $x = 0$ and with the plane of its area parallel to the y-z plane. Find the

(a) spherical **E**- and **H**-field components radiated by the loop in the far zone
(b) directivity of the antenna

5.15. Repeat Problem 5.14 when the plane of the loop is parallel to the x-z plane at $y = 0$.

5.16. Using the computer program of this chapter, compute the radiation resistance and the directivity of a circular loop of constant current with a radius of

(a) $a = \lambda/50$ (b) $a = \lambda/10$ (c) $a = \lambda/4$ (d) $a = \lambda/2$

5.17. A constant current circular loop of radius $a = 5\lambda/4$ is placed on the x-y plane. Find the *two* smallest angles (excluding $\theta = 0^\circ$) where a null is formed in the far-field pattern.

5.18. Design a circular loop of constant current such that its field intensity vanishes only at $\theta = 0^\circ (\theta = 180^\circ)$ and 90°. Find its

(a) radius
(b) radiation resistance
(c) directivity

5.19. Design a constant current circular loop so that its first minimum, aside from $\theta = 0^\circ$, in its far-field pattern is at 30° from a normal to the plane of the loop. Find the

(a) smallest radius of the antenna (in wavelengths)
(b) relative (to the maximum) radiation intensity (in dB) in the plane of the loop

5.20. Design a constant current circular loop so that its pattern has a null in the plane of the loop, and two nulls above and two nulls below the plane of the loop. Find the

(a) radius of the loop
(b) angles where the nulls occur

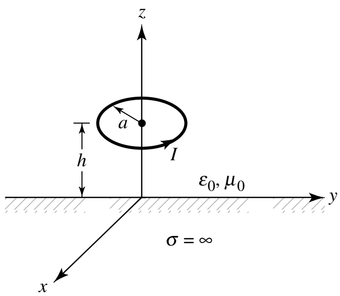
5.21. A constant current circular loop is placed on the x-y plane. Find the far-field position, relative to that of the loop, that a linearly polarized probe antenna must have so that the polarization loss factor (PLF) is maximized.

5.22. A very small ($a \ll \lambda$) circular loop of constant current is placed a distance h above an infinite electric ground plane. Assuming z is perpendicular to the ground plane, find the total far-zone field radiated by the loop when its plane is parallel to the

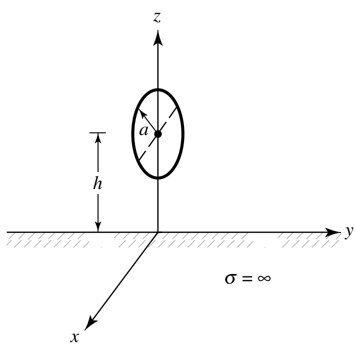
(a) x-z plane
(b) y-z plane

5.23. A very small loop antenna ($a \ll \lambda/30$) of constant current is placed a height h above a flat, perfectly conducting ground plane of infinite extent. The area plane of the loop is parallel to the interface (x-y plane). For far-field observations

(a) find the total electric field radiated by the loop in the presence of the ground plane
(b) all the angles (in degrees) from the vertical to the interface where the total field will vanish when the height is λ
(c) the smallest nonzero height (in λ) such that the total far-zone field exhibits a null at an angle of 60° from the vertical



5.24. A small circular loop, with its area parallel to the x-z plane, is placed a height h above an infinite flat perfectly electric conducting ground plane. Determine
(a) the array factor for the equivalent problem which allows you to find the total field on and above the ground plane
(b) angle(s) θ (in degrees) where the array factor will vanish when the loop is placed at a height $\lambda/2$ above the ground plane



5.25. A small circular loop with its area parallel to the x-z plane is placed at a height h above an infinite perfectly conducting ground plane, as shown in the figure for Problem 5.24. Determine the
(a) array factor for the equivalent problem which will allow you to find the total field *on and above* the ground plane.
(b) two *smallest* heights h (*in* λ) *greater than* $h = 0$ (i.e., $h > 0$) that will form a maximum on the magnitude of the array factor toward $\theta = 0^\circ$.

5.26. For the loop of Problem 5.22(a), find the smallest height h so that a null is formed in the y-z plane at an angle of 45° above the ground plane.

5.27. A small single-turn circular loop of radius $a = 0.05\lambda$ is operating at 300 MHz. Assuming the radius of the wire is $10^{-4}\lambda$, determine the

(a) loss resistance
(b) radiation resistance
(c) loop inductance

Show that the loop inductive reactance is much greater than the loss resistance and radiation resistance indicating that a small loop acts primarily as an inductor.

5.28. Determine the radiation resistance of a single-turn small loop, assuming the geometrical shape of the loop is

(a) rectangular with dimensions a and b $(a, b \ll \lambda)$
(b) elliptical with major axis a and minor axis b $(a, b, \ll \lambda)$

5.29. A one-turn small circular loop is used as a radiating element for a VHF ($f =$ 100 MHz) communications system. The loop is constructed out of a perfect electric conducting wire. The circumference of the loop is $C = \lambda/20$ while the radius of the wire is $\lambda/400$. Determine, using $\sigma = 5.7 \times 10^7$ S/m, the

(a) input resistance of the wire for a single turn.
(b) input reactance of the loop. *Is it inductive or capacitive? Be specific.*
(c) inductance (*in henries*) or capacitance (*in farads*) that can be placed *in series* with the loop at the feed to resonate the antenna at $f = 100$ MHz; choose the element that will accomplish the desired objective.

5.30. Show that for the rectangular loop the radiation resistance is represented by

$$R_r = 31{,}171\left(\frac{a^2b^2}{\lambda^4}\right)$$

while for the elliptical loop is represented by

$$R_r = 31{,}171\left(\frac{\pi^2a^2b^2}{16\lambda^4}\right)$$

5.31 Assuming the direction of the magnetic field of the incident plane wave coincides with the plane of incidence, derive the effective length of a small circular loop of radius a based on the definition of (2-92). Show that its effective length is

$$\boldsymbol{\ell}_e = \hat{\mathbf{a}}_\phi jkS\sin(\theta)$$

where $S = \pi a^2$.

5.32. A circular loop of nonconstant current distribution, with circumference of 1.4λ, is attached to a 300-ohm line. Assuming the radius of the wire is $1.555 \times 10^{-2}\lambda$, find the

(a) input impedance of the loop
(b) VSWR of the system
(c) inductance or capacitance that must be placed across the feed points so that the loop becomes resonant at $f = 100$ MHz.

5.33. A very popular antenna for amateur radio operators is a square loop antenna (referred to as *quad antenna*) whose circumference is one wavelength. Assuming the radiation characteristics of the square loop are well represented by those of a circular loop:

(a) What is the input impedance (real and imaginary parts) of the antenna?

(b) What element (inductor or capacitor), and of what value, must be placed in series with the loop at the feed point to resonate the radiating element at a frequency of 1 GHz?

(c) What is the input VSWR, having the inductor or capacitor in place, if the loop is connected to a 78-ohm coaxial cable?

5.34. Design circular loops of wire radius b, which resonate at the first resonance. Find

(a) four values of a/b where the first resonance occurs (a is the radius of the loop)

(b) the circumference of the loops and the corresponding radii of the wires for the antennas of part (a).

5.35. Using the asymptotic form of (5-59b) for small argument, show that the radiation resistance of (5-64a) for a small loop of uniform current is given by

$$R_r = 20\pi^2(ka)^4 = 20\pi^2\left(\frac{C}{\lambda}\right)^4$$

5.36. Consider a circular loop of wire of radius a on the x-y plane and centered about the origin. Assume the current on the loop is given by

$$I_\phi(\phi') = I_0\cos(\phi')$$

(a) Show that the far-zone electric field of the loop is given by

$$E_\theta = \frac{j\eta ka}{2} I_0 \frac{e^{-jkr}}{r} \frac{J_1(ka\sin\theta)}{ka\sin\theta}\cos\theta\sin\phi$$

$$E_\phi = \frac{j\eta ka}{2} I_0 \frac{e^{-jkr}}{r} J_1{}'(ka\sin\theta)\cos\phi$$

where

$$J_1{}'(x) = \frac{dJ_1(x)}{dx}$$

(b) Evaluate the radiation intensity $U(\theta,\phi)$ in the direction $\theta = 0$ and $\phi = \frac{\pi}{2}$ as a function of ka.