Chapter 2:
Fundamental parameters of antennas

• From Radiation Pattern
  – Radiation intensity
  – Beamwidth
  – Directivity
  – Antenna efficiency
  – Gain
  – Polarization

• From Circuit viewpoint
  – Input Impedance
Chapter 2 : Topics (2)

- Antenna effective length and effective area
- Friis transmission equation
- Radar range equation
- Antenna temperature
Definition of Radiation Pattern

• Once the electromagnetic (EM) energy leaves the antenna, the radiation pattern tells us how the energy propagates away from the antenna.

• Definition:
  Mathematical function or a graphical representation of the radiation properties of an antenna as a function of space coordinates.
Radiation Pattern Example
Radiation Pattern (1)

- Can be classified as:
  - Isotropic, directional and omnidirectional
- Isotropic: Hypothetical antenna having equal radiation in all directions
- Directional: having the property of transmitting or receiving EM energy more effectively in some directions than others
- Omnidirectional: having an essentially nondirectional pattern in a given plane and a directional pattern in any orthogonal plane
Radiation Pattern (2)

- **Principal patterns (or planes):**
  - E-plane: the plane containing the electric field vector and the direction of maximum radiation
  - H-plane: the plane containing the magnetic field vector and the direction of maximum radiation
Radiation Pattern (3)

Omnidirectional
Field Regions

Far field region (Fraunhofer zone)

D=largest antenna dimension

D=largest antenna dimension

Radiating near field region (Fresnel zone)

Reactive near field region
Reactive Near Field Region

- Region surrounding the antenna, wherein the reactive field predominates

For $D > \lambda : R < 0.62 \sqrt{\frac{D^3}{\lambda}}$

For $D < \lambda$ (small antenna): $R < \frac{\lambda}{2\pi}$

$\Rightarrow R < \max[\frac{\lambda}{2\pi}, 0.62 \sqrt{\frac{D^3}{\lambda}}]$
Radiating Near Field Region

- Region between reactive near-field and far-field regions (Fresnel zone)

For $D > \lambda : \frac{2D^2}{\lambda} > R > 0.62\sqrt{\frac{D^3}{\lambda}}$

For $D < \lambda$ (small antenna): $3\lambda > R > \frac{\lambda}{2\pi}$

$\Rightarrow \max[3\lambda, \frac{2D^2}{\lambda}] > R > \max[\frac{\lambda}{2\pi}, 0.62\sqrt{\frac{D^3}{\lambda}}]$

Radiation fields predominate but angular field distribution still depends on distance from antenna
Far Field Region

- Region where angular field distribution is essentially independent of the distance from antenna (Fraunhofer zone)

For $D > \lambda : R > \frac{2D^2}{\lambda}$

For $D < \lambda$ (small antenna): $R > 3\lambda$

$\Rightarrow R > \max[3\lambda, \frac{2D^2}{\lambda}]$
Change of antenna amplitude pattern shape
Spherical coordinate and Solid Angle : Steradian

Measure of solid angle: 1 steradian = solid angle with its vertex at the center of a sphere of radius \( r \) that is subtended by a surface of area \( r^2 \)

\[
dA = r^2 \sin \theta d\theta d\phi = r^2 d\Omega; \\
d\Omega = \sin \theta d\theta d\phi = \text{solid angle}
\]

Quiz: What’s the solid angle subtended by a sphere?
Radiation Power Density

- **Poynting vector = Power density**
  \[ \mathbf{W} = \mathbf{E} \times \mathbf{H} \]

  \( \mathbf{W} \): instantaneous Poynting vector [W/m\(^2\)]
  \( \mathbf{E} \): instantaneous electric field Intensity [V/m]
  \( \mathbf{H} \): instantaneous magnetic field Intensity [A/m]

- **Total power:**
  \[ P = \iiint_S \mathbf{W} \cdot d\mathbf{s} = \iint_S \mathbf{W} \cdot \mathbf{n} \, da \]

  \( P \): instantaneous total power [W]
  \( \mathbf{n} \): unit vector normal to the surface
  \( da \): infinitesimal area of the closed surface [m\(^2\)]
Radiation Power Density (2)

• For time-harmonic EM fields

\[
\vec{E}(x, y, z; t) = \text{Re}[\vec{E}(x, y, z)e^{j\omega t}]
\]

\[
\vec{H}(x, y, z; t) = \text{Re}[\vec{H}(x, y, z)e^{j\omega t}]
\]

• Poynting vector

\[
\vec{S} = \vec{E} \times \vec{H} = \frac{1}{2} \text{Re}[\vec{E} \times \vec{H}^*] + \frac{1}{2} \text{Re}[\vec{E} \times \vec{H}e^{j2\omega t}]
\]

• Time average Poynting vector (average power density or radiation density)

\[
\vec{W}_{av}(x, y, z) = [\vec{S}(x, y, z; t)]_{av} = \frac{1}{2} \text{Re}[\vec{E} \times \vec{H}^*]
\]

\[
\frac{1}{2}
\]

appears because \(\vec{E}, \vec{H}\) fields represent peak values
Radiation Power Density (3)

• Average power radiated power

\[
P_{rad} = P_{av} = \oint_{S} \mathbf{\hat{W}}_{av} \cdot ds = \oint_{S} \mathbf{\hat{W}}_{av} \cdot \mathbf{\hat{n}} da = \frac{1}{2} \oint_{S} \text{Re}[\mathbf{E} \times \mathbf{H}^*] \cdot ds
\]

Example 2.1: The average power density is given by

\[
\mathbf{\hat{W}}_{av} = \hat{r} \mathbf{W}_{r} = \hat{r} A_0 \frac{\sin \theta}{r^2} [W / m^2]
\]

The total radiated power becomes

\[
P_{rad} = \oint_{S} \mathbf{\hat{W}}_{av} \cdot \mathbf{\hat{n}} da
\]

\[
= \int_{0}^{2\pi} \int_{0}^{\pi} (\hat{r} A_0 \frac{\sin \theta}{r^2}) \cdot \hat{r} r^2 \sin \theta d\theta d\phi = \pi^2 A_0 [W]
\]
Radiation Power Density (4)

- For an isotropic antenna

\[ P_{rad} = \iiint_{S} \hat{\mathbf{W}}_{0} \cdot d\mathbf{s} \]

\[ = \int_{0}^{2\pi} \int_{0}^{\pi} [\hat{r}W_{0}(r)] \cdot \hat{r}r^{2} \sin \theta d\theta d\phi = 4\pi r^{2}W_{0} \ [W] \]

The power density is then given by

\[ \hat{\mathbf{W}}_{0} = \hat{r}W_{0} = \hat{r} \frac{P_{rad}}{4\pi r^{2}} \ [W / m^{2}] \]
Radiation Intensity

• **Definition**: The power radiated from an antenna per unit solid angle

\[ U = r^2 W_{rad} \]

- \( U \): radiation intensity [W/unit solid angle]
- \( W_{rad} \): radiation density [W/m\(^2\)]

Total power can be given by

\[ P_{rad} = \iiint_{\Omega} Ud\Omega = \int_{0}^{2\pi} \int_{0}^{\pi} U \sin \theta d\theta d\phi \]
Radiation Intensity (2)

- Radiation intensity is related to the far-zone electric field of antenna

\[ U(\theta, \phi) = \frac{r^2}{2\eta} |\vec{E}(r, \theta, \phi)|^2 \approx \frac{r^2}{2\eta} \left[ |E_\theta(r, \theta, \phi)|^2 + |E_\phi(r, \theta, \phi)|^2 \right] \]

\[ \approx \frac{1}{2\eta} \left[ |E^o_\theta(\theta, \phi)|^2 + |E^o_\phi(\theta, \phi)|^2 \right] \]

\( \vec{E}(r, \theta, \phi) \): far-zone electric-field intensity of the antenna = \( \vec{E}^o(\theta, \phi) e^{-jkr} \)

\( E_\theta, E_\phi \): far-zone electric-field components of the antenna

\( \eta \): intrinsic impedance of the medium (\( \approx 377 \Omega \) in free space)
Example 2.2: The radiation intensity is given by

\[ U = r^2 W_{rad} = A_0 \sin \theta \]

The total radiated power becomes

\[ P_{rad} = \int_0^{2\pi} \int_0^{\pi} U \sin \theta d\theta d\phi \]

\[ = A_0 \int_0^{2\pi} \int_0^{\pi} \sin^2 \theta d\theta d\phi = \pi^2 A_0 [W] \]

For an isotropic antenna

\[ P_{rad} = \iiint_{\Omega} U_0 d\Omega = U_0 \iiint_{\Omega} d\Omega = 4\pi U_0 \]

\[ \Rightarrow U_0 = \frac{P_{rad}}{4\pi} \]
Beamwidth

- Beamwidth is the angular separation between two identical points on opposite site of the pattern maximum
- Half-power beamwidth (HPBW): in a plane containing the direction of the maximum of a beam, the angle between the two directions in which the radiation intensity is one-half value of the beam
- First-Null beamwidth (FNBW): angular separation between the first nulls of the pattern
Beamwidth (2)
Beamwidth (3)

Example 2.3: The normalized radiation intensity of an antenna is represented by

\[ U(\theta) = \cos^2 \theta \ (0 \leq \theta \leq \pi, 0 \leq \phi \leq 2\pi) \]

The angle \( \theta_h \) at which the function equal to half of its maximum can be found by

\[ U(\theta) \big|_{\theta=\theta_h} = \cos^2 \theta = 0.5 \Rightarrow \cos \theta_h = 0.707 \]

\[ \theta_h = \cos^{-1}(0.707) = \frac{\pi}{4} \]

Since the pattern is symmetric with respect to the maximum, HPBW = 2 \( \theta_h = \pi/2 \)

Likewise, FNBW = 2\( \theta_n \) = \( \pi \) since \( U(\theta) \big|_{\theta=\theta_n} = 0 \Rightarrow \theta_n = \cos^{-1}(0) = \frac{\pi}{2} \)
Directivity

- Ratio of radiation intensity in a given direction from the antenna to the average radiation intensity

\[ D(\theta, \phi) = \frac{U(\theta, \phi)}{U_0} = \frac{4\pi U}{P_{rad}} \text{ (dimension-less)} \]

Note that the average radiation intensity equals to the radiation intensity of an isotropic source.
Directivity (2)

Since

\[ P_{rad} = \iiint_{\Omega'} U(\theta', \phi') d\Omega' \]

\[ D(\theta, \phi) = 4\pi \frac{U(\theta, \phi)}{\iiint_{\Omega'} U(\theta', \phi') d\Omega'} \]

\[ D_{\text{max}} = 4\pi \frac{U_{\text{max}}}{\iiint_{\Omega'} U(\theta', \phi') d\Omega'} = \frac{4\pi}{\Omega_{A}} \quad D_{\text{max}}: \text{maximum directivity} \]

\( \Omega_{A} \) is called beam solid angle, and is defined as “solid angle through which all the power of the antenna would flow if its radiation intensity were constant and equal to \( U_{\text{max}} \) for all angles within \( \Omega_{A} \)

\[ \Omega_{A} = \iiint_{\Omega'} \frac{U(\theta', \phi')}{U_{\text{max}}} d\Omega' \Rightarrow P_{rad} = \Omega_{A} U_{\text{max}} \]
Directivity (3)

- If the direction is not specified, it implies the directivity of maximum radiation intensity (maximum directivity) expressed as

\[ D_{\text{max}} = D_0 = \frac{U_{\text{max}}}{U_0} = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} \text{ (dimension - less)} \]
Directivity (4)

Example 2.4: The radial component of the radiated power density of an infinitesimal linear dipole is given by

\[ W_{av} = \hat{r} W_r = \hat{r} A_0 \frac{\sin^2 \theta}{r^2} \text{[W/m}^2\text{]} \]

where \( A_0 \) is the peak value of the power density. The radiation intensity is given by

\[ U = r^2 W_r = A_0 \sin^2 \theta \]

The maximum radiation is directed along \( \theta = \pi/2 \) and \( U_{\text{max}} = A_0 \).

The total radiated power is given by

\[ P_{\text{rad}} = \oiint_{\Omega} U d\Omega = A_0 \int_0^{2\pi} \int_0^{\pi} \sin^2 \theta \sin \phi d\theta d\phi = A_0 \frac{8\pi}{3} \]

Thus,

\[ D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \frac{3}{2} \text{ and } D = D_0 \sin^2 \theta = 1.5 \sin^2 \theta \]
Antenna Efficiency

- The overall antenna efficiency take into the following losses:
  - Reflections because of the mismatch between the transmission line and the antenna
  - Conduction and dielectric losses

\[
e_0 = e_{cd}e_r = e_{cd}(1 - |\Gamma|^2)
\]

- \(e_0\) : total efficiency
- \(e_r\) : reflection (mismatch) efficiency
- \(e_{cd}\) : antenna radiation efficiency = \(e_c e_d\)
- \(e_c, e_d\) : conduction, dielectric efficiencies
- \(\Gamma\) : voltage reflection coefficient at the input terminal
Gain

- It takes into account the efficiency of the antenna as well as its directional properties. (Directivity only measures directional properties.)

\[ G(\theta, \phi) = 4\pi \frac{U(\theta, \phi)}{P_{in}} \]

\[ P_{in} : \text{Power input to antenna;} \]

\[ P_{in} = (1-|\Gamma|^2)P_o = P_{rad} + P_{loss} \]

\( P_{loss} : \text{Ohmic and dielectric power loss} \)

Gain: ratio of radiation intensity in a given direction to the average radiation intensity that would be obtained if all the power input to the antenna were radiated isotropically.
Gain (2)

- Using $e_{cd}$, $P_{rad} = e_{cd}P_{in}$ and

$$G(\theta, \phi) = 4\pi \frac{U(\theta, \phi)}{P_{rad}} e_{cd} = e_{cd} D(\theta, \phi)$$

Relative gain: ratio of power gain in a given direction to the power gain of a reference antenna in the same direction. The power input must be the same for both antennas. If the reference antenna is a lossless isotropic source, then

$$G(\theta, \phi) = 4\pi \frac{U(\theta, \phi)}{P_{in} (\text{lossless isotropic source})}$$
Absolute gain takes into account impedance mismatch losses at the input terminals in addition to losses within antenna.

\[
G_{abs}(\theta, \phi) = 4\pi \frac{U(\theta, \phi)}{P_o} = 4\pi \frac{U(\theta, \phi)}{P_{in}} (1 - |\Gamma|^2)
\]

\[
= 4\pi \frac{U(\theta, \phi)}{P_{rad}} (1 - |\Gamma|^2) e_{cd} = e_0 D(\theta, \phi)
\]
Polarization

- Property of an EM wave describing the time varying direction and relative magnitude of the electric field. The figure traced as a function of time by the tip of the electric field and the sense in which is traced, as observed along direction of propagation.

Let \( E_x(z) = E_{xo} e^{j \phi_x} e^{jkz} \),
\[ E_y(z) = E_{yo} e^{j \phi_y} e^{jkz} \] where \( E_{xo}, E_{yo} \geq 0 \)

\( E_x(z; t) = E_{xo} \cos(\omega t + kz + \phi_x) \)
\( E_y(z; t) = E_{yo} \cos(\omega t + kz + \phi_y) \)

**Wave propagating in \(-z\) direction**

\( e^{j\omega t} \) time dependence
Polarization (2)

A. Linear Polarization

(i) \( E_{xo} = 0 \) or \( E_{yo} = 0 \)

or

(ii) \( \delta = \phi_y - \phi_x = n\pi \)

where \( n = 0, \pm 1, \pm 2, \ldots \)

\( \gamma = \tan^{-1}\left( \frac{E_{yo}}{E_{xo}} \right) \), \( 0 \leq \gamma \leq \frac{\pi}{2} \)

\( \delta, \gamma \) determine polarization state
Polarization (3)

B. Circular Polarization

(i) \( E_{xo} = E_{yo} = E_o \Rightarrow \gamma = \tan^{-1}(1) = \pi / 4 \)

and

(ii) \( \delta = \phi_y - \phi_x = \begin{cases} \left(2n + \frac{1}{2}\right)\pi; & \text{CW/RCP} \\ -\left(2n + \frac{1}{2}\right)\pi; & \text{CCW/LCP} \end{cases} \)

where \( n = 0,1,2,\ldots \)

Note that the sense of rotation is observed along the direction of propagation.
Polarization Ellipse & Sense of Rotation for Antenna Coordinate System

**Sense Of Rotation**

- **Right hand**
- **Clockwise**
Polarization (4)

Example: RCP

\[ \gamma = \pi / 4, \delta = \pi / 2 \]

\[ \mathcal{E}_x(z; t) = E_o \cos(\omega t + kz + \phi_x) \]

\[ \mathcal{E}_y(z; t) = E_o \cos(\omega t + kz + \phi_x + \pi / 2) \]

\[ = - E_o \sin(\omega t + kz + \phi_x) \]
C. Elliptic Polarization

- A wave is elliptically polarized if it is not linearly or circularly polarized.
- Linear and circular polarization are special cases of elliptic polarization.

To have elliptic polarization:

1. Field must have two orthogonal linear components.
2. The two components can be of the same or different magnitude.
C. Elliptic Polarization

(i) If \( \delta = \phi_y - \phi_x = \left\{ \begin{array}{l} \left( 2n + \frac{1}{2} \right) \pi; \text{ CW/REP} \\ -\left( 2n + \frac{1}{2} \right) \pi; \text{ CCW/LEP} \end{array} \right. \)

where \( n = 0, 1, 2, \ldots \) AND \( E_{xo} \neq E_{yo} \)

(ii) If \( \delta = \phi_y - \phi_x \neq \pm \frac{n\pi}{2} \left\{ \begin{array}{l} > 0; \text{ CW/REP} \\ < 0; \text{ CCW/LEP} \end{array} \right. \)

where \( n = 0, 1, 2, \ldots \) FOR \( \forall E_{xo}, \forall E_{yo} \)
Polarization (7)

Axial Ratio (AR) = \frac{\text{Major Axis}}{\text{Minor Axis}} \quad ; \quad 1 \leq \text{AR} \leq \infty

\tau = \frac{\pi}{2} - \frac{1}{2} \tan^{-1}\left( \frac{2E_{xo}E_{yo}}{E_{xo}^2 - E_{yo}^2} \cos \delta \right)

[Diagram of polarization ellipse with labeled axes and angles]
Polarization Loss Factor (PLF)

- Electric field of incoming wave
  \[ \vec{E}_w = \hat{\rho}_w E_i \]
- Electric field of receiving antenna
  \[ \vec{E}_a = \hat{\rho}_a E_a \]

where

\( \hat{\rho}_w \): unit vector of the wave
\( \hat{\rho}_a \): polarization vector

\[ \text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = |\cos \psi_p|^2 \]
(dimensionless)
PLF example

LCP wave: $\delta=-\pi/2, \phi_x=0 \Rightarrow \phi_y=-\pi/2$

$$E_x = E_o e^{j\phi_x} e^{j k z}; E_y = E_o e^{j\phi_y} e^{j k z}$$

$$\mathbf{E} = (\hat{x}E_o + \hat{y}E_o e^{-j \pi/2}) e^{j k z} = (\hat{x} - j \hat{y}) E_o e^{j k z} = \hat{\rho}_w \sqrt{2} E_o e^{j k z}$$

where

$$\hat{\rho}_w = \frac{\hat{x} - j \hat{y}}{\sqrt{2}}$$

If the antenna is also LCP, $\hat{\rho}_a = \hat{\rho}_w^*$

$$\text{PLF} = \left| \frac{\hat{x} - j \hat{y}}{\sqrt{2}} \cdot \frac{\hat{x} + j \hat{y}}{\sqrt{2}} \right|^2 = 1 = 0 \text{ dB}$$

If the antenna is RCP, $\hat{\rho}_a = \frac{\sqrt{2}}{\hat{x} - j \hat{y}}$

$$\text{PLF} = \left| \frac{\hat{x} - j \hat{y}}{\sqrt{2}} \cdot \frac{\hat{x} - j \hat{y}}{\sqrt{2}} \right|^2 = 0$$
Input Impedance

Impedance presented by the antenna at its terminal

\[ Z_A(\omega) = R_A(\omega) + jX_A(\omega) \quad [\Omega] \]

- **Transmitting case**

\[ R_A = R_r + R_{loss} \]

- **\( R_r \)**: Radiation resistance

- **\( R_{loss} \)**: Loss resistance (ohmic, dielectric)

\[ R_{loss} = R_{ohmic} + R_d \]

\[ Z_g = R_g + jX_g, V_g = I_g (Z_g + Z_A) \]

- \( V_g, I_g \) are peak values

Maximum power delivered to the antenna occurs when conjugate matched:

\[ R_A = R_g, X_A = -X_g \]
Input Impedance (2)

When conjugate matched:

\[ I_g = \frac{V_g}{2R_A} \Rightarrow P_{rad} = \frac{1}{2} |I_g|^2 \quad R_r = \frac{|V_g|^2}{8} \frac{R_r}{(R_r + R_{loss})^2} \]

Radiated Power

\[ P_{loss} = \frac{1}{2} |I_g|^2 \quad R_{loss} = \frac{|V_g|^2}{8} \frac{R_{loss}}{(R_r + R_{loss})^2} \]

Power loss to heat

\[ P_g = \frac{1}{2} |I_g|^2 \quad R_g = \frac{|V_g|^2}{8} \frac{1}{R_r + R_{loss}} \]

Power loss in \( R_g \)

NOTE: \[ P_g = P_{rad} + P_{loss} \]
Input Impedance (3)

Power supplied by generator when conjugate matched:

\[
P_s = \text{Re}\left[ \frac{1}{2} V_g I_g^* \right] = \frac{1}{2} V_g \frac{V_g^*}{2R_A} = \frac{|V_g|^2}{4R_A} = \frac{|V_g|^2}{4R_g}
\]

\[
P_g = \frac{1}{2} P_s
\]

\[
P_{in} = P_{rad} + P_{loss} = \frac{1}{2} P_s
\]

\[
e_{cd} = \frac{P_{rad}}{P_{in}} = \frac{R_r}{R_r + R_{loss}}
\]

If \( R_{loss} = 0 \Rightarrow P_{loss} = 0, P_{rad} = P_{in}, e_{cd} = 1 \)
Receiving Antenna

Impedance presented by the antenna at its terminal

\[ Z_A = R_A + jX_A, \quad Z_T = R_T + jX_T \]

under conjugate matched condition

\[ R_T = R_A = R_r + R_{loss}, \quad X_T = -X_A \]

\[ I_T = \frac{V_T}{2R_T} \Rightarrow P_T = \frac{1}{2} |I_T|^2, \quad R_T = \frac{|V_T|^2}{8R_T} \]

\[ V_T, I_T \text{ are peak values} \]

Power delivered to load

\[ P_{scatt} = \frac{1}{2} |I_T|^2, \quad R_r = \frac{|V_T|^2}{8} \frac{R_r}{(R_r + R_{loss})^2} \]

Power scattered or re-radiated
Receiving Antenna (2)

Power supplied by generator when conjugate matched:

\[ P_{\text{loss}} = \frac{1}{2} |I_T|^2 \]
\[ R_{\text{loss}} = \frac{|V_T|^2}{8} \frac{R_{\text{loss}}}{(R_r + R_{\text{loss}})^2} \]

\[ P_T = P_{\text{scatt}} + P_{\text{loss}} \]

Power lost to heat

\[ P_{\text{c}} = \frac{1}{2} \text{Re}[V_T I_T^*] = \frac{|V_T|^2}{4R_T} \]

captured/collected power

\[ P_T = \frac{1}{2} P_{\text{c}} \]

\[ P_{\text{scatt}} + P_{\text{loss}} = \frac{1}{2} P_{\text{c}} \]

\[ P_{\text{c}} = P_{\text{scatt}} + P_{\text{loss}} + P_T \]
Antenna equivalent Area

- Used to describe the power capturing characteristics of an antenna when a wave impinges on it

\[ A_e(\theta, \phi) = \text{effective area (aperture) in direction } (\theta, \phi) \]

\[ = \frac{\text{Power available at terminals of receiving antenna}}{\text{Incident power flux density from direction } (\theta, \phi)} \]

\[ A_e = \frac{P_T}{W_i} = \frac{|I_T|^2 R_T}{2 W_i} \quad [\text{m}^2] \]

\( P_T \): power delivered to load \( R_T \)

\( W_i \): power density of incident wave
Antenna Equivalent Area (2)

\[ A_e = \frac{|V_T|^2}{2W_i} \left[ \frac{R_T}{(R_r + R_{loss} + R_T)^2 + (X_A + X_T)^2} \right] \]

Under conjugate matched condition:

\[ A_{em} = \frac{|V_T|^2}{8W_i R_T} = \frac{|V_T|^2}{8W_i} \frac{1}{R_r + R_{loss}} \]

\[ A_s = \frac{|V_T|^2}{8W_i} \frac{R_r}{(R_r + R_{loss})^2} \]

- \( A_e \): effective area, which when multiplied by the incident power density, is equal to power delivered to load \( R_T \).
- \( A_s \): scattering area, which when multiplied by incident power density, is equal to the scattered or re-radiated power.
Antenna Equivalent Area (3)

Under conjugate matched condition:

\[
A_{\text{loss}} = \frac{|V_T|^2}{8W_i} \frac{R_{\text{loss}}}{(R_r + R_{\text{loss}})^2}
\]

\[
A_c = \frac{|V_T|^2}{4W_i R_T} = \frac{|V_T|^2}{8W_i} \frac{R_r + R_{\text{loss}} + R_T}{(R_r + R_{\text{loss}})^2}
\]

\[A_c = A_e + A_s + A_{\text{loss}}\]

\[A_{\text{loss}} : \text{loss area, which when multiplied by the incident power density, is equal to power delivered to load } R_{\text{loss}}\]

\[A_c : \text{Captured area, which when multiplied by incident power density, is equal to the total power captured by antenna}\]
Antenna Equivalent Area (4)

Example 2.5: a uniform plane wave is incident upon very short dipole, whose radiation resistance is $R_r=80(\pi l/\lambda)^2$. Assume that $R_{\text{loss}} = 0$, the maximum effective area reduces to

$$A_{em} = \frac{|V_T|^2}{8W_iR_r}$$

Since the dipole is very short, the induced current can be assumed to be constant and of uniform phase. The induced voltage is

$$V_T = El$$

For a uniform plane wave, the incident power density is given by

$$W_i = \frac{E^2}{2\eta}$$

thus

$$A_{em} = \frac{(El)^2}{8(E^2 / 2\eta)(80\pi^2 l^2 / \lambda^2)} = \frac{3\lambda^2}{8\pi} = 0.119\lambda^2$$
Vector Effective Length

- Vector effective length (or height) is a quantity used to determine the voltage induced on the open-circuit terminals of an antenna when a wave impinges on it. It is a far-field quantity.

\[ \vec{l}_e(\theta, \phi) = \hat{\theta} l_\theta(\theta, \phi) + \hat{\phi} l_\phi(\theta, \phi) \quad [\text{m}] \]

\[ V_{oc} = \vec{E}^i \cdot \vec{l}_e \]

\[ \vec{E}_a = \hat{\theta} E_\theta + \hat{\phi} E_\phi = -j \eta \frac{k I_{in}}{4 \pi r} e^{-jkr} \vec{l}_e \]

Example 2.6: The electric field of a short dipole is given by

\[ \vec{E}_a = \hat{\theta} j \eta \frac{k I_{in}}{8 \pi r} e^{-jkr} \sin \theta \quad \rightarrow \quad \vec{l}_e = -\hat{\theta} \frac{l}{2} \sin \theta \]
Effective area & Directivity

If antenna #1 were isotropic, its radiated power density at a distance $R$ would be

$$ W_0 = \frac{P_t}{4\pi R^2} $$

where $P_t$ is the total radiated power. Because of the directivity, the actual power density becomes

$$ W_t = W_0 D_t = \frac{P_t D_t}{4\pi R^2} $$

The power collected by the antenna would be

$$ P_r = W_t A_r = \frac{P_t D_t A_r}{4\pi R^2} $$

or

$$ D_t A_r = \frac{P_r}{P_t} 4\pi R^2 \quad (1) $$
Effective area & Directivity\(^{(2)}\)

If antenna \#2 is used as a transmitter, 1 as a receiver, and the medium is linear, passive and isotropic, one obtains

\[ D_r A_t = \frac{P_r}{P_t} 4\pi R^2 \quad (2) \]

From (1) and (2),

\[ \frac{D_t}{A_t} = \frac{D_r}{A_r} \]

Increasing the directivity of an antenna increases its effective area:

\[ \frac{D_{0r}}{A_{tm}} = \frac{D_{0r}}{A_{rm}} \]

If antenna \#1 is isotropic,

\[ A_{tm} = \frac{A_{rm}}{D_{0r}} \]
Effective area & Directivity (3)

For example, if antenna #2 is a short dipole, whose effective area is $3\lambda^2/8\pi$ and directivity is 1.5, one obtains

$$A_{tm} = \frac{A_{rm}}{D_0} = \frac{3\lambda^2}{2} \frac{2}{8\pi} = \frac{\lambda^2}{4\pi}$$

and

$$A_{rm} = D_0 A_{tm} = D_0 \frac{\lambda^2}{4\pi}$$

In general, maximum effective area of any antenna is related to its maximum directivity by

$$A_{em} = \frac{\lambda^2}{4\pi} D_0$$

If there’re conduction-dielectric loss, polarization loss and mismatch:

$$A_{em} = \frac{\lambda^2}{4\pi} D_0 e_{cd} (1 - |\Gamma|^2) |\hat{\rho}_w \cdot \hat{\rho}_a|^2$$
Friis Transmission Equation

- The Friis transmission equation relates the power received to the power transmitted between two antennas separated by a distance $R > 2D^2/\lambda$.

Power density at distance $R$ from the transmitting antenna:

$$W_t = \frac{P_t G_t(\theta_t, \phi_t)}{4\pi R^2} = \frac{e_i P_t D_t(\theta_t, \phi_t)}{4\pi R^2}$$

The effective area of the receiving antenna:

$$A_r = e_r D_r(\theta_r, \phi_r) \frac{\lambda^2}{4\pi}$$

The amount of power collected by the receiving antenna:

$$P_r = e_r D_r(\theta_r, \phi_r) \frac{\lambda^2}{4\pi} W_t = e_i e_r \frac{\lambda^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r) P_t}{(4\pi R)^2} |\hat{\rho}_t \cdot \hat{\rho}_r|$$
The ratio of the received to the input power:

\[
\frac{P_r}{P_t} = e_t e_r \frac{\lambda^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{(4\pi R)^2} |\hat{\rho}_t \cdot \hat{\rho}_r|
\]

or

\[
\frac{P_r}{P_t} = e_{cdt} e_{cdr} (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2) \frac{\lambda^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{(4\pi R)^2} |\hat{\rho}_t \cdot \hat{\rho}_r|
\]

For reflection and polarization-matched antennas aligned for maximum directional radiation and reception:

\[
\frac{P_r}{P_t} = \left( \frac{\lambda}{4\pi R} \right)^2 G_{0t} G_{0r}
\]
Radar Cross Section

- Radar cross section or echo area ($\sigma$) of a target is defined as the area intercepting that amount of power which, when scattered isotropically, produces at the receiver a density which is equal to that scattered by the actual target.

$$\sigma = \lim_{R \to \infty} \left[ 4\pi R^2 \frac{W_s}{W_i} \right] = \lim_{R \to \infty} \left[ 4\pi R^2 \frac{|\vec{E}^s|^2}{|\vec{E}^i|^2} \right] = \lim_{R \to \infty} \left[ 4\pi R^2 \frac{|\vec{H}^s|^2}{|\vec{H}^i|^2} \right]$$

$\sigma$ : radar cross section [m$^2$]

$R$ : observation distance from target [m]

$W_i, W_s$ : incident, scattered power density [W/m$^2$]

$\vec{E}^i, \vec{E}^s$ : incident, scattered electric field [V/m]

$\vec{H}^i, \vec{H}^s$ : incident, scattered magnetic field [A/m]
<table>
<thead>
<tr>
<th>Object</th>
<th>Typical RCSs [22]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RCS (m²)</td>
</tr>
<tr>
<td>Pickup truck</td>
<td>200</td>
</tr>
<tr>
<td>Automobile</td>
<td>100</td>
</tr>
<tr>
<td>Jumbo jet airliner</td>
<td>100</td>
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<tr>
<td>Large bomber or</td>
<td>40</td>
</tr>
<tr>
<td>commercial jet</td>
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<td>Cabin cruiser boat</td>
<td>10</td>
</tr>
<tr>
<td>Large fighter aircraft</td>
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<tr>
<td>Small fighter aircraft</td>
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<tr>
<td>or four-passenger jet</td>
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<td>missile</td>
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<tr>
<td>Insect</td>
<td>0.00001</td>
</tr>
<tr>
<td>Advanced tactical fighter</td>
<td>0.000001</td>
</tr>
</tbody>
</table>
Radar Range Equation

- The radar range equation relates the power delivered to the receiver load to the power transmitted by an antenna, after it has been scattered by a target with a radar cross section of $\sigma$.

The amount of captured power at the target with the distance $R_1$ from the transmitting antenna:

$$P_c = \sigma W_t = \sigma \frac{P_l G_t(\theta_t, \phi_t)}{4\pi R_1^2} = \frac{e_t P_l D_t(\theta_t, \phi_t)}{4\pi R_1^2}$$

The scattered power density:

$$W_s = \frac{P_c}{4\pi R_2^2} = e_t \sigma \frac{P D_t(\theta_t, \phi)}{(4\pi R_1 R_2)^2}$$
Radar Range Equation (2)

The amount of power delivered to the load:

\[ P_r = A_r W_s = e_t e_r \sigma \frac{P_t D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{4\pi} \left( \frac{\lambda}{4\pi R_1 R_2} \right)^2 \]

Thus,

\[ \frac{P_r}{P_t} = A_r W_s = e_t e_r \sigma \frac{D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{4\pi} \left( \frac{\lambda}{4\pi R_1 R_2} \right)^2 \]

With polarization loss:

\[ \frac{P_r}{P_t} = A_r W_s = e_{ct} e_{cd} (1 - |\Gamma_t|^2) (1 - |\Gamma_r|^2) \sigma \frac{D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{4\pi} \left( \frac{\lambda}{4\pi R_1 R_2} \right)^2 |\hat{\rho}_w \cdot \hat{\rho}_r|^2 \]

For reflection and polarization-matched antennas aligned for maximum directional radiation and reception:

\[ \frac{P_r}{P_t} = \sigma \frac{G_{0t} G_{0r}}{4\pi} \left( \frac{\lambda}{4\pi R_1 R_2} \right)^2 \]
Brightness Temperature

- Energy radiated by an object can be represented by an equivalent temperature known as Brightness Temperature $T_B$ (K).

\[
T_B(\theta, \phi) = \varepsilon(\theta, \phi)T_m = \left(1 - |\Gamma|^2\right)T_m
\]

where

- $\varepsilon$: emissivity (dimensionless) $0 \leq \varepsilon \leq 1$
- $T_m$: molecular (physical) temperature (K)
- $\Gamma(\theta, \phi)$: reflection coefficient of the surface for the polarization of wave

Example Ground 300 K
Antenna Temperature

- Energy radiated by various sources appears at antenna terminal as antenna temperature, given by

\[
T_A = \frac{\int_0^{2\pi} \int_0^\pi T_B(\theta, \phi)G(\theta, \phi)\sin \theta d\theta d\phi}{\int_0^{2\pi} \int_0^\pi G(\theta, \phi)\sin \theta d\theta d\phi}
\]

where

- \(T_A\): antenna temperature (effective noise temperature of the antenna radiation resistance; K)
- \(G(\theta, \phi)\): gain (power) pattern of the antenna
**Noise Power**

- Assuming no losses or other contributions between antenna & receiver, noise power transferred to receiver:

\[ P_r = k T_A \Delta f \]

where

- \( P_r \): antenna noise power (W)
- \( k \): Boltzmann’s constant (\(1.38 \times 10^{-23}\) J/K)
- \( T_A \): antenna temperature (K)
- \( \Delta f \): bandwidth (Hz)
System Noise Power Model

- Assume antenna & transmission line are maintained at certain temperature, and transmission line is lossy, then the model below can be used to include all contributions.
Antenna Temperature (2)

- Effective antenna temperature at receiver terminals:

\[ T_a = T_A e^{-2\alpha l} + T_{AP} e^{-2\alpha l} + T_0 (1 - e^{-2\alpha l}) \]

where

- \( T_a \) : antenna temperature at receiver terminals (K)
- \( T_A \) : antenna noise temperature at antenna terminals (K)
- \( T_{AP} \) : antenna temperature at antenna terminals due to physical temperature (K)
- \( T_0 \) : physical temperature of transmission line (K)
- \( \alpha \) : attenuation constant of transmission line (Np/m)
- \( e_A \) : thermal efficiency of antenna (dimensionless)
- \( l \) : length of transmission line (m)
System Noise Power

- noise power transferred to receiver: $P_r = kT_a \Delta f$
- If there’s thermal noise in receiver:

\[
P_s = k(T_a + T_r)\Delta f = kT_s \Delta f
\]

where

- $P_s$: system noise power (W)
- $T_a$: antenna noise temperature at receiver terminals
- $T_r$: receiver noise temperature at receiver terminals
- $T_s = T_a + T_r$: effective system noise temperature at receiver terminals
Ex 2.16 Effective antenna temp = 150 K. Antenna is maintained at 300 K and has thermal efficiency 99%. It is connected to a receiver through 10-m waveguide (loss = 0.13 dB/m, temp = 300 K) Find effective antenna temperature at receiver terminals.

\[ T_{AP} = (e_A^{-1} - 1)T_P = (0.99^{-1} - 1)300 = 3.03 \]

\[ \alpha (\text{Np/m}) = \alpha (\text{dB/m})/8.68 = 0.0149, \alpha l = 0.149 \]

\[ T_a = T_Ae^{-2\alpha l} + T_{AP}e^{-2\alpha l} + T_0(1 - e^{-2\alpha l}) \]

\[ = 150e^{-2(0.149)} + 3.03e^{-2(0.149)} + 300(1 - e^{-2(0.149)}) \]

\[ = 190.904 \text{ K} \]